NECOM20002

**Forecasting in Economics and Business**

**Tutorial 7 Solutions**

1. Let be a set of realizations of from an i.i.d. sequence in which each is characterized by a Poisson distribution function:
2. Using words, provide an explanation of the parameter and the role it plays in determining the shape of the distribution function.

**The Poisson distribution is often used to model the number of times an event occurs in a given interval of time and space. The parameter represents the average number of events that occur in that interval and can be thought of as the event rate. also happens to be the mean and variance of the Poisson random variable and thus determines the locus and spread of the probability mass across its possible values.**

1. Write down the likelihood and log-likelihood function associated with the set of realizations .

**The likelihood function is given by,**

**Which means that the log-likelihood function will be given by,**

1. Given the realizations , what is the maximum likelihood estimate of the parameter ?

**The maximum likelihood estimate of the parameter is obtained by taking the first derivative of the log likelihood function and setting it equal to zero and then solving for .**

**Therefore we have that,**

1. Using the rpois() function in ***R***, generate a set of 500 independent realizations from a Poisson random variable where . Using these realizations, compute the maximum likelihood estimate of the parameter Does your estimate conform to expectations?

**This simply amounts to computing the sample mean of the 500 observations. It should be quite close to 2.**

1. Repeat part d an additional 499 times. You will have 500 samples of 500 observations from which you will obtain 500 estimates of the parameter Plot a histogram of the estimates and discuss its shape. (***Hint: Try writing a loop in R:*** <https://www.r-bloggers.com/how-to-write-the-first-for-loop-in-r/>)

**This should produce a histogram that looks quite close to a normal curve that is centered around 2 and has a variance of !**

1. Suppose that you are analyzing a time series model that behaves according to an ARMA(1,1) process,
2. Given the information set , write down the expressions for the one-step, two-step and -step ahead forecasts.

**The one-step ahead forecast will be given by**

**The two-step ahead forecast will be given by**

**The -step ahead forecast will be given by**

1. Write down the expressions for the one-step, two-step and -step ahead forecast errors and their associated variances. What happens to the forecast error variance when

**The one-step ahead forecast error and forecast error variance will be given by**

**The two-step ahead forecast error and forecast error variance will be given by**

**The -step ahead forecast error and forecast error variance will be given by**

**If the process is covariance stationary and invertible, we have that and . The forecast error variance will grow as but it will be bounded.**

1. Using the data contained in *tute7.csv*, estimate an ARMA(1,1) model in ***R*** and use the estimates and the formulas that you derived in part b to compute -step ahead 95% interval forecasts for . Verify that your computations are correct by computing the forecasts using forecast() function. Describe what happens to the interval forecasts as increases.

**See R code.**